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Thermal-ignition analysis in boundary-layer flows

By C. K. LAW AND H. K. LAW

Department of Mechanical Engineering and Astronautical Sciences,
Northwestern University, Evanston, Illinois 60201

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The weakly reactive state leading to the ignition of a steady laminar boundary-layer flow of a combustible mixture over a hot, isothermal, non-permeable, non-catalytic flat plate is studied both numerically and using matched asymptotic analysis in the realistic limit of large activation energy. It is shown that the flow consists of a locally similar diffusive–reactive region next to the plate and a non-similar diffusive–convective region outside it; that the analytic solution obtained reproduces the lower half of the S-shaped ignition–extinction response curve such that ignition is expected to occur when a suitably defined Damköhler number, which increases with the streamwise distance, reaches unity; and that at the point of ignition the heat transfer from the wall vanishes identically. An explicit expression for the minimum distance for ignition to occur is also derived.

1. Introduction

The ignition of a combustible gas flow by either a hot body or another hot gas stream is of interest to such practical problems as flame stabilization in combustors and the initiation of accidental fires and explosions. When the flow Reynolds number is high, ignition is frequently achieved within the boundary-layer region, where the mixture becomes appreciably heated. Therefore the phenomena of interest are those of chemically reacting boundary-layer flows.

One of the major difficulties involved with fundamental studies of this problem is the lack of similarity in the flow properties. This is because, whereas self-similar solutions abound for chemically inert boundary-layer flows, similarity is frequently destroyed in the presence of finite rate chemical reactions. In fact the stagnation-point boundary-layer flow is probably the only one in which the similarity is preserved (Law 1978).

Various analytical techniques have been devised to obtain approximate solutions to the system of highly complex partial differential equations governing the flow. Prominent among them are Marble & Adamson's (1954) series expansion technique, which employs the streamwise co-ordinate s as the expansion parameter, and Dooley's (1957) iteration technique, which treats terms with explicit dependence on s as of lower order and hence described by the solution of the previous iteration.

With these approximations each partial differential equation is reduced to a hierarchy of linear, inhomogeneous, ordinary differential equations, which are then solved numerically. However, since the accuracy of these approximations depends on s being small, as s increases more expansion terms or iteration steps are needed for convergence. This not only rapidly multiplies the numerical effort, but the accuracy

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of the solution is also substantially diminished owing to accumulated errors in the previous solutions.

Examples of other types of approximation are Lees' (1958) local-similarity approximation of highly cooled hypersonic boundary layers and Marble & Adamson's (1954) integral method of tracing the flame development in the plane mixing-layer flow. These special techniques have been extensively reviewed by, for example, Hayes & Probstein (1959), Dorrance (1962) and Chung (1965).

In the present study we aim to analyse the structure of chemically reacting boundary-layer flows both numerically and also by using the newly developed technique of matched asymptotic analysis as applied to practical combustion systems, which are usually characterized by reactions with large activation energies (Williams 1971). We shall at present restrict our study to the weakly reactive states leading to ignition, and to systems which admit self-similar solutions in the inert limit such that the absence of similarity is strictly caused by the gas-phase reactions. In particular we have as our model problem the laminar flow of a combustible mixture over an isothermal, non-permeable, non-catalytic flat plate. Since this is probably the simplest flow configuration exhibiting reaction-induced non-similarity, the structure of the flow, particularly the question of similarity, can be studied in greater detail without being unduly complicated by non-essential factors.

As will be subsequently demonstrated, the small parameter of expansion for the present system is $\epsilon = \tilde{T}_w^2/\tilde{T}_a$, where $\tilde{T}_w$ and $\tilde{T}_a$ are the non-dimensional wall temperature and activation energy respectively. In the limit of small ϵ, the flow field consists of a locally similar diffusive–reactive region next to the plate and a non-similar diffusive–convective region outside it. The identification of this flow structure not only is physically interesting but also facilitates the mathematical manipulations such that the problem can be completely solved analytically. In particular, we have derived an explicit expression for the ignition distance, which is a parameter of practical importance.

Finally, we note a very recent paper (Berman & Ryazantsev 1978) which has also employed the matched asymptotic technique to analyse the same problem. However, although the general analytic technique adopted is the same, the two studies differ in several important aspects. For example, in the present work we take the physical viewpoint that effects due to chemical reactions are manifested as perturbations to the self-similar boundary-layer profiles in the inert limit, and subsequently investigate and elaborate on situations under which local similarity can still be preserved. We also show that the conventional boundary-layer parameter $f'(\eta)$ [see equations (8) and (11)] not only is a more natural and convenient choice for the transverse independent variable, but also may result in a more accurate representation of the inner solution. Furthermore, the present study provides a rigorous analysis of the outer solution, and thereby the matching relation between the inner and outer solutions. The validity and accuracy of the asymptotic approximation have also been numerically investigated here.

In the next section the governing equations and the problem of interest are defined. In § 3 numerical solutions are presented and compared with those obtained by assuming complete local similarity in the flow field. In § 4 the matched asymptotic analysis is conducted and the final solutions derived.

2. Formulation

The problem of interest is the steady laminar boundary-layer flow of a combustible mixture with velocity u_∞, temperature T_∞, pressure P and species mass fractions $Y_{i\infty}$ over an isothermal, non-permeable, non-catalytic flat plate of temperature T_w. It is assumed that the reaction between the fuel F and the oxidizer O leading to the formation of some product can be represented by a one-step overall irreversible reaction

$$\nu_O[O]+\nu_F[F]\rightarrow\nu_P[\text{product}] \tag{1}$$

with a reaction rate proportional to

$$C_O^p C_F^q T^r \exp(-E/R^0 T). \tag{2}$$

The conservation equations of momentum, species i and energy are then respectively (Williams 1965)

$$f'''+ff''=0, \tag{3}$$

$$\mathscr{L}_1\{\tilde{Y}_i\}=-\mathscr{L}_1\{\tilde{T}\}=(2s/\rho u_\infty)\omega, \tag{4}$$

where a prime indicates $d/d\eta$ and the operator $\mathscr{L}_1$ is defined as

$$\mathscr{L}_1\equiv\frac{\partial^2}{\partial\eta^2}+f\frac{\partial}{\partial\eta}-2sf'\left(\frac{\partial}{\partial s}\right). \tag{5}$$

Here s and η are Howarth–Dorodnitsyn variables related to the physical co-ordinates x and y parallel and normal to the surface, respectively, according to

$$s=x, \quad \eta=\left(\frac{u_\infty}{2x\rho_\infty\mu_\infty}\right)^{\frac{1}{2}}\int_0^y\rho(x,y')\,dy', \tag{6),(7}$$

$f(s,\eta)$ is related to the stream function $\psi(x,y)$ through

$$f(s,\eta)=\psi(x,y)/(2x\rho_\infty\mu_\infty u_\infty)^{\frac{1}{2}}, \tag{8}$$

ω is the chemical production term

$$\omega=\left(\frac{B\nu_F W_F}{W_O^p W_F^q}\right)\left(\frac{P\overline{W}}{R^0}\right)^{p+q}T^{r-p-q}\,Y_O^p Y_F^q\exp(-E/R^0T), \tag{9}$$

while $\tilde{T}=c_pT/Q$, $\tilde{Y}_i=(\nu_F W_F/\nu_i W_i)Y_i$, T is the temperature, P the pressure, ρ the density, u the x velocity, Y the mass fraction, C the molar concentration, c_p the specific heat, μ the viscosity coefficient, W the molecular weight, $\overline{W}$ an average molecular weight, B the frequency factor, E the activation energy, ν the stoichiometric molar coefficient, Q the chemical heat release per unit mass of fuel consumed, R^0 the universal gas constant and the subscripts ∞ and w respectively designate the free stream and the wall. Furthermore, in deriving (4), we have assumed that viscous heating is negligible, that the Prandtl and Schmidt numbers are unity, and that the product $\rho\mu$ is a constant.

Equation (4) is in the conventional form adopted in the study of chemically reacting boundary-layer flows. For each value of i, this equation consists of four terms representing effects due to transverse diffusion, transverse convection, streamwise convection

and reaction. We have, however, found that the problem can be more naturally and conveniently described by the new independent variables.

$$\zeta = x\left(\frac{2B\sigma^p}{u_\infty}\right)\left(\frac{\nu_F W_F}{W_O^p W_F^q}\right)\left(\frac{P\overline{W}}{R^0}\right)^{p+q-1}\left(\frac{Q}{c_p}\right)^{r-p-q+1} \tag{10}$$

and

$$\xi = f'(\eta), \tag{11}$$

where σ is the stoichiometric ratio of oxidizer mass to fuel mass.

Expressing (4) in terms of ζ and ξ, we have

$$\mathscr{L}_2\{\tilde{Y}_i\} = -\mathscr{L}_2\{\tilde{T}\} = \zeta(f'')^{-2}\tilde{Y}_O^p\tilde{Y}_F^q\tilde{T}^{(r-p-q+1)}\exp(-\tilde{T}_a/\tilde{T}), \tag{12}$$

where

$$\mathscr{L}_2 = \frac{\partial^2}{\partial\xi^2} - \frac{2\zeta\xi}{(f'')^2}\frac{\partial}{\partial\zeta} \tag{13}$$

and $T_a = E/R^0$. The above transformation suppresses the transverse convection term such that (12) can be interpreted as consisting of three terms representing (transverse) diffusion, (streamwise) convection and reaction. It is also of interest to note that, rather than appearing independently, x (or s) really belongs to the group represented by ζ, which can be identified as some kind of Damköhler number.

Equations (3) and (12) are to be solved subject to the boundary conditions

$$f(0) = f'(0) = 0, \quad f'(\infty) = 1, \tag{14}$$

$$(\partial\tilde{Y}_i/\partial\xi)_{\xi=0} = 0, \quad \tilde{Y}_i(\zeta, 1) = \tilde{Y}_{i\infty}, \tag{15}$$

$$\tilde{T}(\zeta, 0) = \tilde{T}_w, \quad \tilde{T}(\zeta, 1) = \tilde{T}_\infty. \tag{16}$$

We shall also assume for simplicity that the initial profiles, at $\zeta = 0$, are similar.

Equations (12) and (13) show that the non-similar nature of the flow is a direct result of the occurrence of chemical reaction in that in its absence the $\partial/\partial\zeta$ terms will vanish identically under the present initial and boundary conditions. However, when a reaction is occurring, its effects are perpetually being felt in the streamwise direction owing to the presence of ζ in the reaction term. The flow then ceases to be similar.

It is, however, reasonable to expect that, since chemical reaction is weak over most of the pre-ignition region, the streamwise variations of the flow properties are likely to be minimal. In other words terms involving $\partial/\partial\zeta$ are expected to be locally small relative to the diffusion and reaction terms and hence, in accordance with the concept of local similarity, may be neglected. Consequently variations in ζ are manifested parametrically, through the presence of ζ in the reaction term, rather than differentially. The governing equations are then simplified to ordinary differential equations.

The assumption of local similarity will obviously break down in the vicinity of the ignition point, where flow properties change rapidly in the streamwise direction, although the associated error in estimating the ignition point is expected to be small. Furthermore a more accurate description in the ignition region within the context of boundary-layer theory may not be meaningful because the presence of large gradients in the streamwise direction violates the basic assumption of a boundary layer.

It may also be noted that, since by invoking local similarity the history in the streamwise direction is completely ignored, the accuracy of the solution obviously depends on how strongly the flow depends on its history.

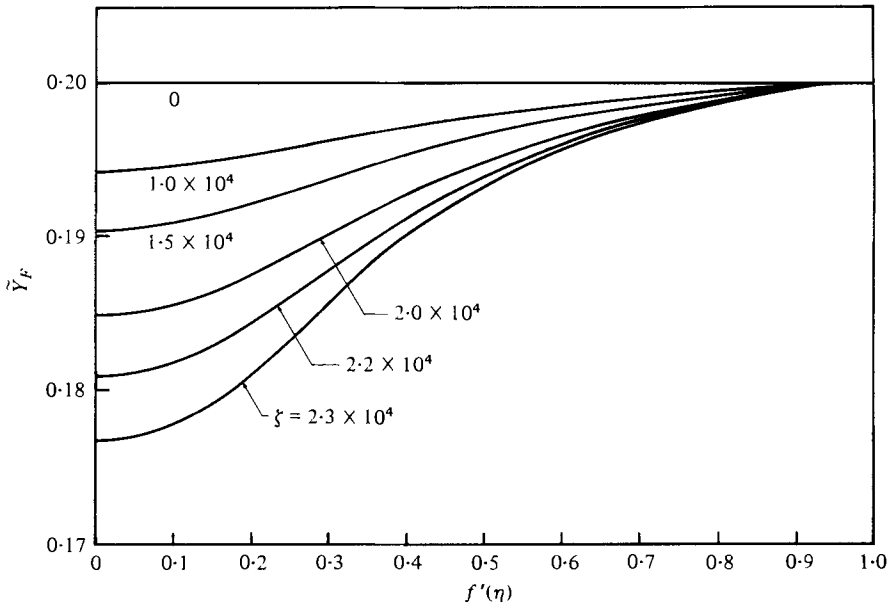

FIGURE 1. Non-similar species profiles.

In view of these discussions, it is of interest to investigate in general the characteristics of the flow, particularly the development of the reaction zone leading to ignition and the extent to which local similarity is valid. We have therefore numerically integrated (3) and (12) with and without the $\partial/\partial\zeta$ terms.

3. Numerical solutions

In solving (3) and (12), a second-order-correct numerical scheme is employed, with implicit determination in the η direction and marching in the ζ direction. The grid sizes are 0·02 for η and variable for ζ, although never greater than $0{\cdot}02\zeta_I$, where the subscript I designates the ignition point. For simplicity we have also assumed that $p = q = r = 1$, and that the mixture is stoichiometric, with $\tilde{Y}_{O\infty} = \tilde{Y}_{F\infty}$. In obtaining the results to be presented in figures 1 to 3, we have used the values $\tilde{T}_a = 0{\cdot}25$, $\tilde{T}_w = 0{\cdot}025$, $\tilde{T}_\infty = 0{\cdot}0075$ and $\tilde{Y}_{F\infty} = 0{\cdot}2$, which correspond to $T_a = 10^4$ °K, $T_w = 10^3$ °K, $T_\infty = 3 \times 10^2$ °K, $c_p = 0{\cdot}25$ cal/g °K and $Q = 10^4$ cal/g.

Figures 1 and 2 show the species and temperature profiles as functions of both η and $\xi = f'(\eta)$ for different locations ζ along the plate. At the leading edge $\tilde{Y}_F = \tilde{Y}_{F\infty}$ and $\tilde{T}$ varies linearly with ξ [see (19)] as specified by the initial conditions. As ζ increases, chemical reaction is initiated and leads to depletion of the species concentration and an increase of the gas temperature. For small values of ζ the hot wall is the most reactive state and heat is supplied from the wall to the gas. However, at about $\zeta = 1{\cdot}5 \times 10^4$ the transfer ceases and for higher values of ζ there is sufficient chemical heat generated in the gas that the temperature maximum is shifted away from the wall. The chemical reaction can now be considered to be self-sustaining. Therefore the location where heat transfer at the wall vanishes can be approximately identified as the ignition point (Toong 1957; Sharma & Sirignano 1970).

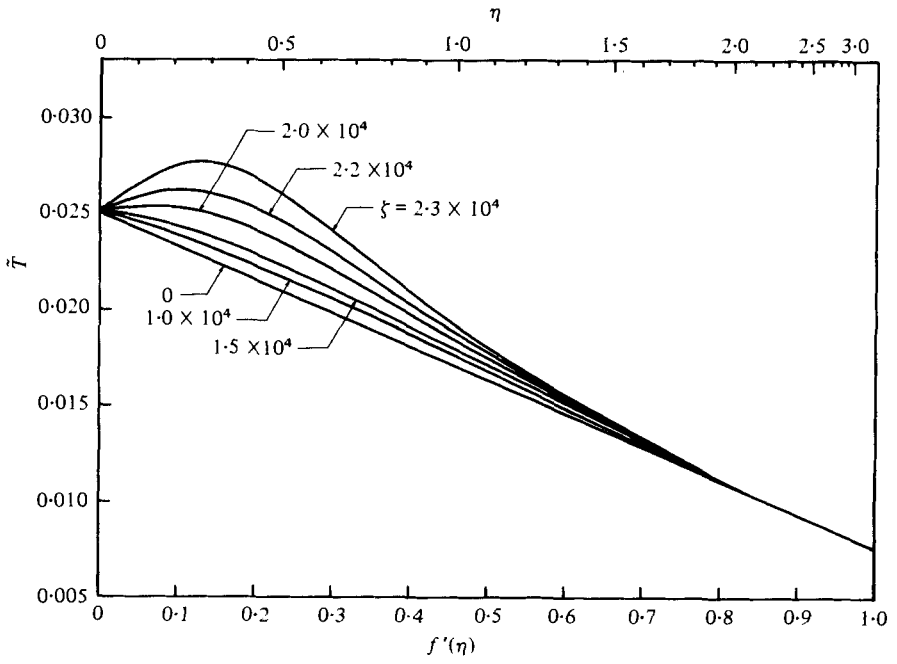

FIGURE 2. Non-similar temperature profiles.

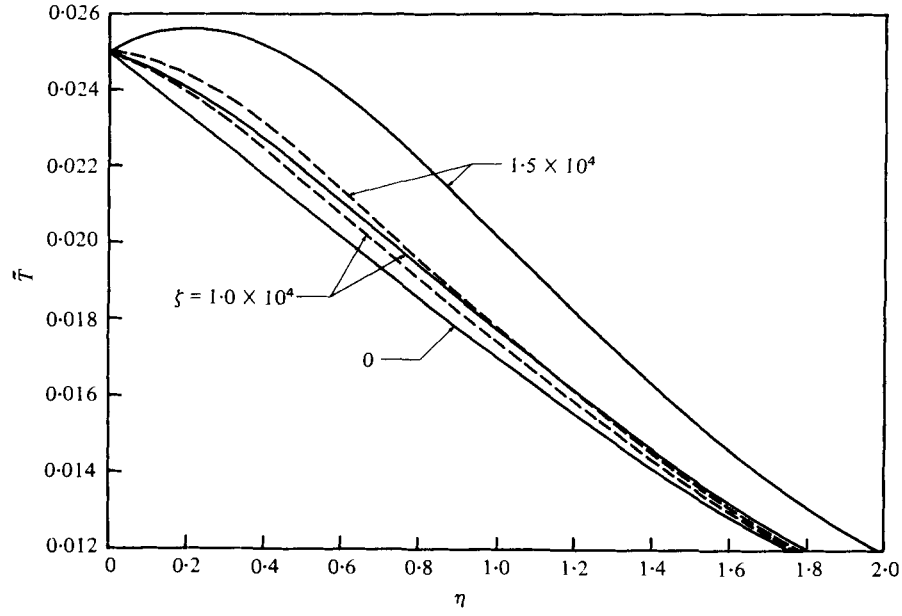

FIGURE 3. Comparison between non-similar and locally similar temperature profiles. –– non-similar; ——, locally similar.

				$\zeta_I(\times 10^4)$		
$\tilde{T}_w$	$\tilde{T}_\infty$	$\tilde{Y}_{F\infty}$	$\tilde{T}_a$	(i)	(ii)	(iii)
0·0250	0·0075	0·2	0·250	1·55	1·37	0·743
0·0255	0·0075	0·2	0·250	1·31	1·16	0·621
0·0250	0·0100	0·2	0·250	1·19	1·02	0·546
0·0250	0·0075	0·1	0·250	6·79	6·59	2·97
0·0250	0·0075	0·2	0·245	1·25	1·11	0·596

TABLE 1. Predicted ignition distances ζ_I for different system parameters. (i) Numerical solution of non-similar flow. (ii) Numerical solution assuming locally similar flow. (iii) Analytic solution.

Figure 3 compares the temperature profiles given by the exact, non-similar solution and the approximate, locally similar solution. Since streamwise convective transport is neglected in the locally similar treatment, higher temperatures are predicted. The agreement between the two profiles is generally close until ζ exceeds about 10^4, at which substantial differences at short distances away from the wall occur. However, agreement in the vicinity of the wall is always good before ignition is achieved. In particular, using the state of zero heat transfer at the wall as the ignition criterion, the non-similar and locally similar solutions respectively predict the ignition distances ζ_I to be $1{\cdot}55 \times 10^4$ and $1{\cdot}37 \times 10^4$.

We have further investigated the sensitivity of the predicted ignition distance by systematically varying one of $\tilde{T}_w$, $\tilde{T}_\infty$, $\tilde{T}_a$ and $\tilde{Y}_{F\infty}$ (table 1). As expected, the system is least sensitive to $\tilde{T}_\infty$ and, owing to the Arrhenius factor, most sensitive to $\tilde{T}_a$ and next $\tilde{T}_w$. Table 1 also shows that, whereas the locally similar solution always predicts smaller values of ζ_I, the difference from the non-similar results is generally less than 15% and hence the agreement can be considered to be very good.

The above results show that the locally similar solution can accurately describe the flow characteristics near the wall, particularly the ignition distance. In the following we shall perform an asymptotic analysis of the governing equations. In so doing we are able to delineate the characteristics of the flow field as a whole; in particular it will also be demonstrated that the flow near the wall is indeed locally similar.

4. Asymptotic analysis

Frozen solution

The present analysis capitalizes on the concept that for combustion systems of practical interest the activation energy is usually large. In the limit $\tilde{T}_a \to \infty$, the flow field is completely frozen and hence self-similar. Equation (12) becomes

$$d^2\tilde{Y}_{if}/d\xi^2 = d^2\tilde{T}_f/d\xi^2 = 0, \tag{17}$$

whose solution is

$$\tilde{Y}_{if} = \tilde{Y}_{i\infty}, \quad \tilde{T}_f = \tilde{T}_w - \beta\xi, \tag{18),(19}$$

where $\beta = \tilde{T}_w - \tilde{T}_\infty$ is a heat trasfer parameter and the subscript f designates the frozen solution.

For large but finite values of $\tilde{T}_a$, chemical reaction is expected to occur first near the hot wall since it has the highest temperature in the flow field. However, at a short

distance away from the wall the slight decrease in temperature is sufficient to freeze the reaction owing to the temperature-sensitive Arrhenius factor. Hence the flow field is expected to consist of an inner reactive region and an outer frozen region. These two regions will now be separately analysed and matched.

Inner, diffusive–reactive, region

In this region the existence of weak chemical activities is expected to modify the flow properties from their frozen values by an amount of order ϵ. Hence we assume an inner solution of the form

$$\tilde{T}_{\text{in}} = \tilde{T}_f + \epsilon\theta(\tau, \chi) + \ldots, \tag{20}$$

$$\tilde{Y}_{i,\text{in}} = \tilde{Y}_{i\infty} + \epsilon\phi(\tau, \chi) + \ldots, \tag{21}$$

where the stretched inner variables are defined as

$$\tau = (\zeta_I - \zeta)/\epsilon, \quad \chi = \beta\xi/\epsilon. \tag{22}$$

It is worth mentioning that, since the inner region extends over small values of the transverse independent variable and since $f'(\eta) < \eta$, the adoption of $\xi = f'(\eta)$ rather than η as the independent variable to be stretched is believed to result in a more accurate representation of the inner solution.

Substituting (20)–(22) into (12), it can be shown that the perturbed temperature $\theta(\tau, \chi)$ is governed by

$$\frac{\partial^2\theta}{\partial\chi^2} = -\epsilon^2 \frac{2\zeta_I\chi}{[\beta f''(0)]^2}\frac{\partial\theta}{\partial\tau} - \frac{\epsilon\zeta_I}{[\beta f''(0)]^2}\tilde{Y}_{O\infty}^{p}\tilde{Y}_{F\infty}^{q}\tilde{T}_w^{r-p-q+1}\exp(-\tilde{T}_a/\tilde{T}_w)\exp(\theta-\chi), \tag{23}$$

where $f''(0) = 0{\cdot}4696$. This equation shows that, since the reaction term has to be of the same order as the diffusion term, the convection term is then of order ϵ^2 and hence can be safely neglected. The exceptionally small magnitude of the convection term is due to two factors. First, since gradients are very steep in the thin reaction region, the diffusion rate is generally one order larger than the convection rate. Secondly, for the present case the convective transport is further slowed because the transport velocity ξ is only of order ϵ near the wall. These combined effects give the weak ϵ^2 dependence.

By neglecting the convection term, we have therefore established that the flow in the inner region is locally similar and is diffusive–reactive in nature. Hence, for general $\zeta < \zeta_I$, (23) becomes

$$d^2\theta/d\chi^2 = -\tfrac{1}{2}\Delta\exp(\theta-\chi), \tag{24}$$

where

$$\Delta = \frac{2\epsilon\zeta}{[\beta f''(0)]^2}\tilde{Y}_{O\infty}^{p}\tilde{Y}_{F\infty}^{q}\tilde{T}_w^{r-p-q+1}\exp(-\tilde{T}_a/\tilde{T}_w), \tag{25}$$

is the relevant Damköhler number for the present system.

Equation (24) is to be solved subject to the boundary condition

$$\theta(0) = 0 \tag{26}$$

at the wall and a second condition obtained through matching with the outer solution.

Finally, it may be noted that in writing down (20) and (21) we have assumed that the expansion is regular in ϵ. However, a recent study by Liñán & Williams (1978) shows that, for the mathematically similar problem of the ignition of a semi-infinite

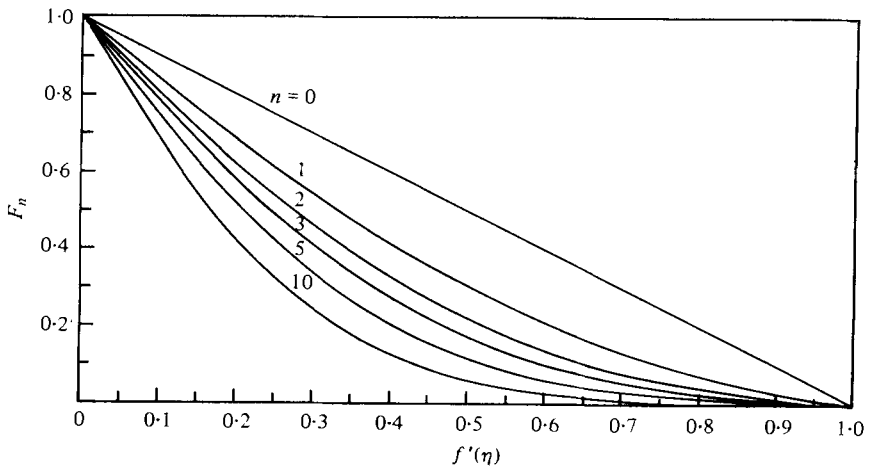

FIGURE 4. The function F_n characterizing the inert flow with varying wall temperature.

reactive solid after a step increase in the surface temperature, the expansion is not regular in that an additional term proportional to $\epsilon \ln \epsilon$ appears when interactions between the preheating and ignition stages are properly accounted for. We have elected not to include this interaction term because the present results are able adequately to describe most of the essential physics without requiring too much mathematical manipulation. Furthermore, as emphasized previously, a more rigorous analysis involving the structure of the ignition region does not appear to be justified for the present problem because the boundary-layer approximation breaks down in this region.

Outer, diffusive–convective, region

In the outer region chemical reaction is expected to be frozen to all orders. It is, however, not possible to neglect the $\partial/\partial\zeta$ term in (12) since at the boundary with the inner region the temperature varies with ζ. The flow is then non-similar, i.e. diffusive–convective in nature, and is governed by

$$\mathscr{L}_2\{\tilde{T}_{\text{out}}\} = 0. \tag{27}$$

Therefore the problem of interest for the outer region is essentially that of an inert flat-plate boundary-layer flow with a varying wall temperature. Several solutions to this problem exist, although we find the one by Chapman & Rubesin (1949) particularly useful for the present application. In their study the wall temperature is assumed to be expressible as a power series in ζ. Through separation-of-variable techniques the final solution is found to be

$$\tilde{T}_{\text{out}} = \tilde{T}_\infty + \beta(1-\xi) + \sum_{n=1}^{\infty} c_n \zeta^n F_n(\xi), \tag{28}$$

where $F_n(\xi)$ is the solution of

$$\frac{d^2F_n}{d\xi^2} - \frac{2n\xi}{(f'')^2} F_n = 0, \quad F_n(0) = 1, \quad F_n(1) = 0 \tag{29}$$

and the sum in (28) is expected to be of the order ϵ. It is obvious that the solution of (29) for small values of ξ is

$$F_n(\xi) \to 1 - \alpha_n \xi \quad \text{as} \quad \xi \to 0, \tag{30}$$

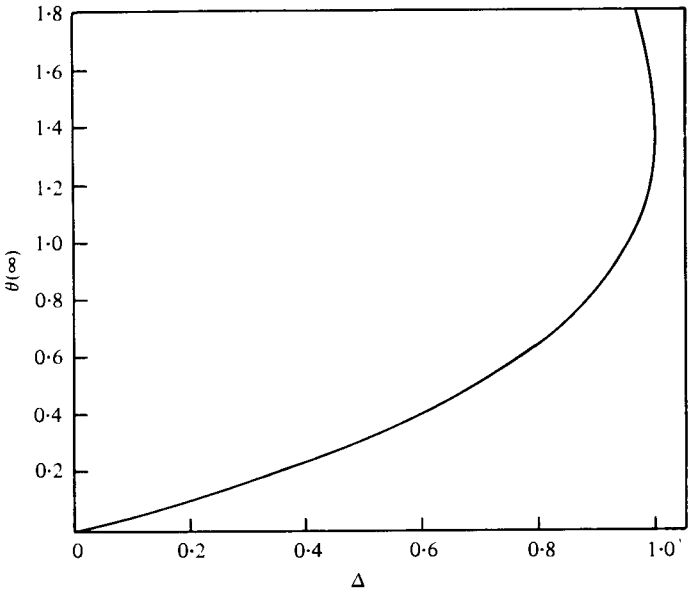

FIGURE 5. Lower half of the characteristic S-curve showing the maximum perturbed temperature as a function of the Damköhler number Δ.

where α_n is the initial, constant slope. Equation (29) has also been solved numerically (figure 4). The results further confirm that $F_n(\xi)$ varies linearly with ξ as $\xi \to 0$.

Matching

By matching the inner solution (20) with the outer solution (28), in the limit $\chi \to \infty$, we have

$$\lim_{\chi\to\infty} \tilde{T}_w + \epsilon[\theta(\chi)-\chi] = \lim_{\chi\to\infty} \tilde{T}_\infty + \beta(1-\epsilon\chi/\beta) + \sum_{n=1}^{\infty} c_n \zeta^n (1-\epsilon\alpha_n\chi/\beta). \tag{31}$$

This yields

$$\theta(\infty) = \left(\sum_{n=1}^{\infty} c_n \zeta^n\right)\Big/\epsilon, \quad (d\theta/d\chi)_\infty = 0. \tag{32), (33}$$

Equation (33) is the additional boundary condition needed to solve for $\theta(\chi)$ in (24). Once $\theta(\chi)$ has been found, the temperature coefficients c_n in the outer region can be determined by expanding $\theta(\infty)$ in powers of $\Delta \sim \zeta$ and equating terms of the same power in (32).

Final solution

It can be shown (Law 1978) that the solution to (24), (26) and (33) is

$$\theta(\chi) = \chi + \ln\left\{\frac{1}{\Delta}\left[1-\left(\frac{\Lambda e^{\pm\chi}-1}{\Lambda e^{\pm\chi}+1}\right)^2\right]\right\} \tag{34}$$

where

$$\Lambda = \frac{1+(1-\Delta)^{\frac{1}{2}}}{1-(1-\Delta)^{\frac{1}{2}}}. \tag{35}$$

Equation (34) shows that for $\Delta < 1$ two solutions exist whereas for $\Delta > 1$ no

solution exists. This behaviour indicates the existence of some critical phenomenon, namely ignition as a result of thermal runaway in the present case, when $\Delta = 1$. This can be illustrated more clearly from a plot (figure 5) of $\theta(\infty)$ *vs.* Δ, with

$$\theta(\infty) = \ln\left\{\frac{4}{\Delta}\left[\frac{1 \mp (1-\Delta)^{\frac{1}{2}}}{1 \pm (1-\Delta)^{\frac{1}{2}}}\right]\right\}. \tag{36}$$

Figure 5 shows that we have obtained the lower half of the S-shaped ignition–extinction response curve (Fendell 1965), $\Delta \geqslant 1$ being the criterion for ignition to take place. Hence using (25), the minimum distance for ignition to occur is

$$\zeta_I = \frac{[\beta f''(0)]^2}{2\epsilon \tilde{Y}_{O\infty}^{p} \tilde{Y}_{F\infty}^{q}} \tilde{T}_w^{p+q-r-1} \exp(\tilde{T}_a/\tilde{T}_w), \tag{37}$$

or in terms of the physical distance x,

$$x_I = \left\{\frac{u_\infty [\beta f''(0)]^2}{4\epsilon B Y_{O\infty}^{p} Y_{F\infty}^{q}}\right\}\left(\frac{W_O^p W_F^q}{\nu_F \overline{W}_F}\right)\left(\frac{R^0}{\overline{P}\overline{W}}\right)^{p+q-1} T_w^{p+q-r-1} \exp(T_a/T_w). \tag{38}$$

Table 1 shows the ignition distance ζ_I as predicted by (37). It is seen that the analytical values are generally smaller than the numerical results for the locally similar solutions by about a factor of two. This is typical of the accuracy of the first-order approximation using large activation energy asymptotes. For example Bush & Fendell's (1970) first-order estimate on the laminar flame propagation rate shows similar deviation from the numerical value, although close agreement is obtained by including the second-order solution.

The heat transfer at the wall is given by

$$(\partial T/\partial y)_{y=0} \sim -(1-\Delta)^{\frac{1}{2}}, \tag{39}$$

which shows that heat is transferred from the wall to the gas during the weakly reactive states until the ignition point $\Delta = 1$ is reached. Therefore to the present order of accuracy the adiabaticity ignition criterion is identical with the thermal runaway ignition criterion. A small but finite distance between these two states will result when higher-order effects are accounted for in the analysis, as was done by Liñán & Williams (1978) for the transient ignition of a solid.

5. Conclusions

Through matched asymptotic analysis in the realistic limit of large activation energy, it is revealed that the structure of weakly reactive flat-plate boundary-layer flow consists of a locally similar diffusive–reactive region next to the hot plate and a non-similar diffusive-convective region outside it. A suitably defined Damköhler number Δ characterizes the continuous increase in the chemical activities as the combustible mixture flows downstream; in particular, it is shown that ignition is expected to occur when Δ reaches unity, at which the heat transfer from the wall also ceases. This ignition criterion produces an explicit expression for the ignition distance, rendering possible its *a priori* estimation and also assessment of the relative importance and sensitivity of the various system parameters in effecting ignition.

Finally, it may also be emphasized that, within the constraint of a one-step overall reaction, the assumed reaction mechanism is quite general, allowing for arbitrary

reaction orders with respect to the reactant concentrations and both power-law and Arrhenius-factor dependence on the temperature. This further enhances the utility of the present ignition criterion.

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